Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

GEOMETRY.

415. Proposed by W. D. CAIRNS, Oberlin, Ohio.

For three points P_1, P_2 and P_3 on a straight line

$$P_1P_2 = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2},$$

$$P_2P_3 = \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2 + (z_3 - z_2)^2},$$

$$P_1P_3 = \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2 + (z_3 - z_1)^2},$$

while $P_1P_2 + P_2P_3 = P_1P_3$; prove directly that the sum of the first two radical expressions equals the third.

SOLUTION BY THE PROPOSER.

Introducing a parameter notation, the equation of the straight line $P_1P_2P_3$ may be written

$$\frac{x - x'}{a} = \frac{y - y'}{b} = \frac{z - z'}{c} = \lambda,$$

or $x = x' + \lambda a$, $y = y' + \lambda b$, etc.; whence, for the three given points (as indicated by subscripts),

$$x_1 = x' + \lambda_1 a, \quad y_1 = y' + \lambda_1 b, \quad \text{etc.};$$

$$x_2 = x' + \lambda_2 a, \quad \text{etc.}, \quad \text{etc.};$$

$$x_3 = x' + \lambda_3 a, \quad \text{etc.}, \quad \text{etc.}$$

The substitution of this notation in the given radical expressions gives them the form

$$(\lambda_2 - \lambda_1)\ \sqrt{a^2 + b^2 + c^2},$$

$$(\lambda_3 - \lambda_2)\ \sqrt{a^2 + b^2 + c^2},$$

$$(\lambda_3 - \lambda_1)\ \sqrt{a^2 + b^2 + c^2}.$$

Whence the desired result is evident.

Also solved by H. C. FEEMSTER, R. M. MATHEWS, A. M. HARDING, JAMES A. BULLARD, and C. A. BARNHART.

416. Proposed by W. H. BUSSEY, University of Minnesota.

Let AB be a diameter of any circle, and let C be the third vertex of the equilateral triangle of which AB is one side. Divide AB into six equal parts and let P denote the second point of division from A (*i. e.*, $AP = \frac{1}{3}AB$). Draw the straight line CP and produce it to meet the circle in R. Prove that AR is one-sixth of the circumference.

SOLUTION BY E. L. BROWN, Denver, Colorado.

Let O be the center of circle. Draw OD perpendicular to CP. In right triangle COP, $PO = r/3$, $CO = r\sqrt{3}$, $CP = 2r/3\sqrt{7}$, $PD = r/6\sqrt{7}$, $CD = 9r/2\sqrt{7}$, $OD^2 = 3r^2/28$. In right triangle ODR, $DR = 5r/2\sqrt{7}$.

$$PR = DR - DP = r\sqrt{7}/3 = \tfrac{1}{2}CP.$$

But $PA = \frac{1}{2}PB$. Therefore triangles APR and CPB are similar. Hence $AR = \frac{1}{2}BC = r$.

Also solved by S. LEFSCHETZ, A. L. MCCARTY, GEORGE N. BAUER, HERMON L. SLOBIN, BARNEM LIBBY, RICHARD MORRIS, E. S. INGHAM, and HARVEY MANN.